Industrial Process Control MDP 454

“If you have a smart project, you can say "I'm an engineer"”

Lecture 9

Staff boarder

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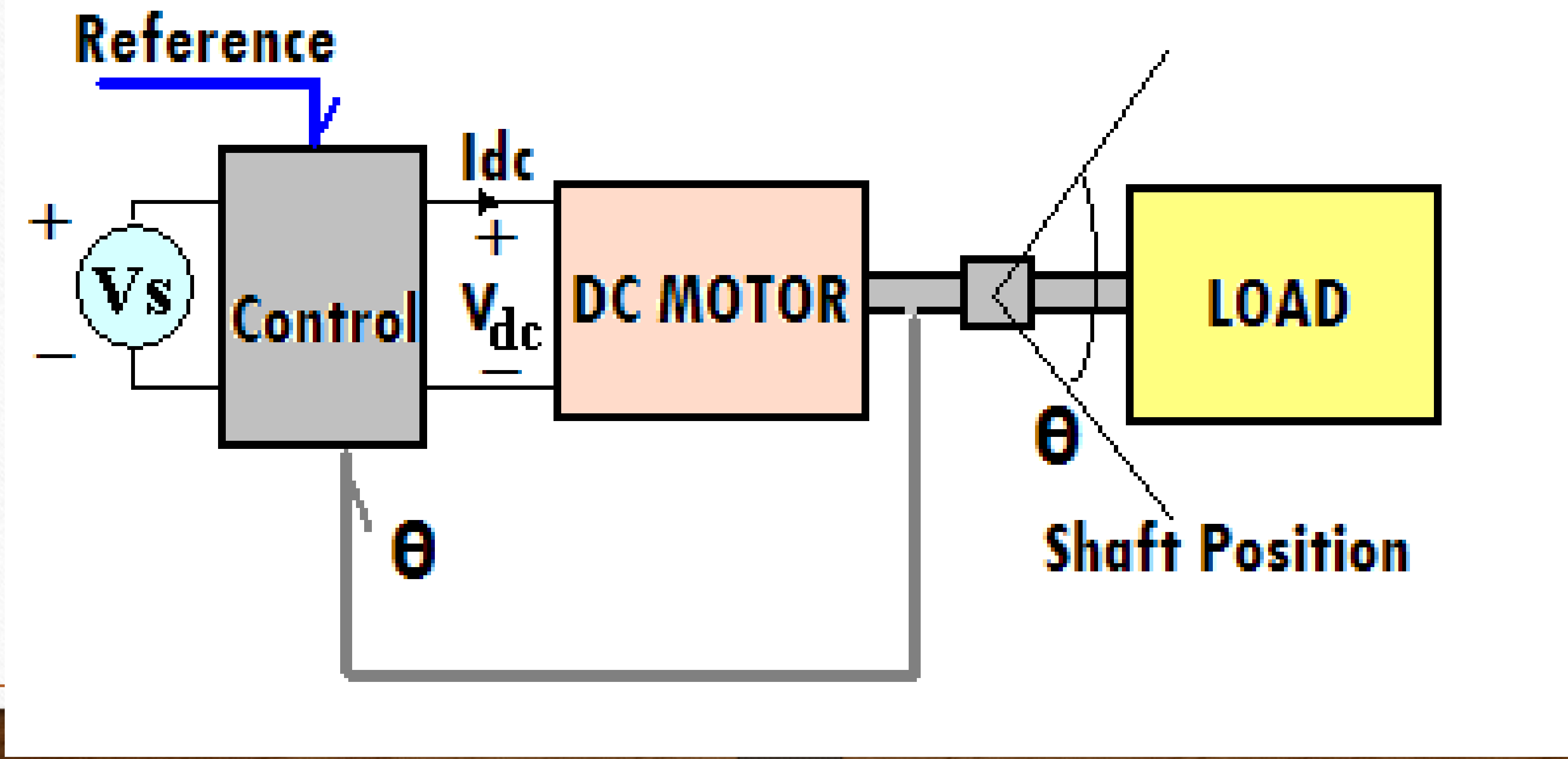
Industrial Process Control
MDP 454

- **Lecture aims:**
  - Be familiar with the PID controller as a key element of many feedback systems.
  - Be capable of designing a controller to meet desired specifications using frequency response methods.

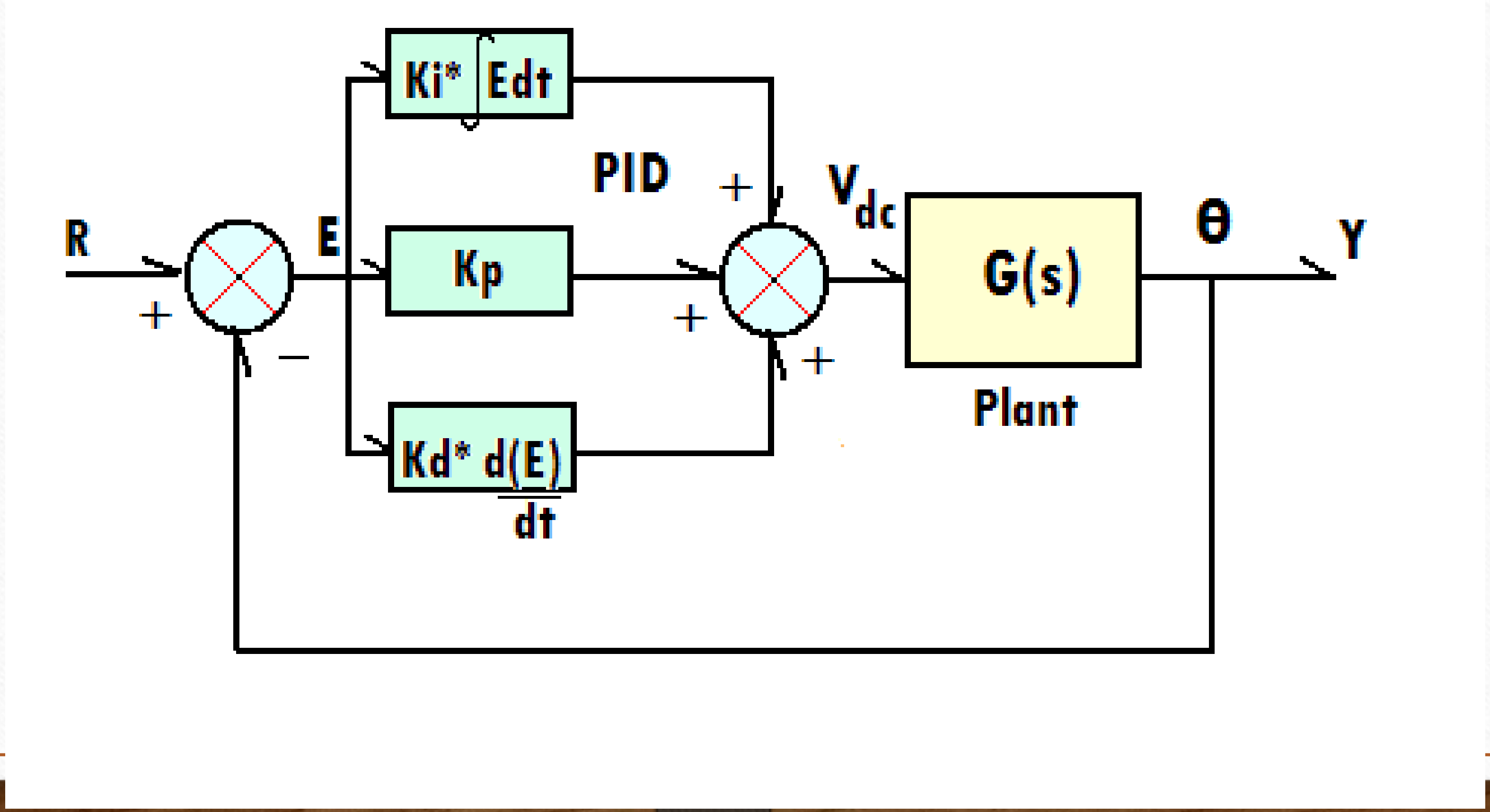
PID Control

- A 2nd Order SISO System with Input to Control Shaft Position:



PID Control

PID Block Diagram:



PID Control

PID Mathematically:

- Consider the input error variable, e(t):
  - Let $p(t) = Kp*e(t)$ {p proportional to e (mag)}
  - Let $i(t) = Ki*\int e(t)dt$ {i integral of e (area)}
  - Let $d(t) = Kd* de(t)/dt$ {d derivative of e (slope)}

AND let $Vdc(t) = p(t) + i(t) + d(t)$

Then in Laplace Domain:

$$Vdc(s) = [Kp + 1/s\ Ki + s\ Kd]\ E(s)$$

Basic Control Modes

Proportional Control

Next we consider the three basic control modes starting with the simplest mode, *proportional control.*

In feedback control, the objective is to reduce the error signal to zero where

$$e(t) = y_{sp}(t) - y_m(t) \qquad (8\text{-}1)$$

and

$e(t)$ = error signal

$y_{sp}(t)$ = set point

$y_m(t)$ = measured value of the controlled variable (or equivalent signal from the sensor/transmitter)

Basic Control Modes

Proportional Control

Although Eq. 8-1 indicates that the set point can be time-varying, in many process control problems it is kept constant for long periods of time.

For proportional control, the controller output is proportional to the error signal,

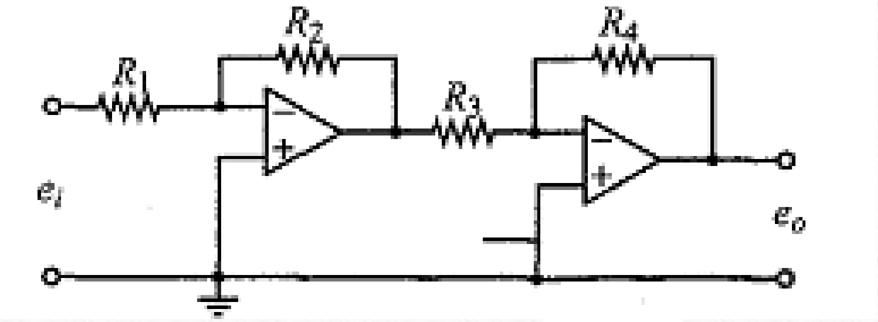
$$p(t) = \bar{p} + K_c e(t) \qquad (8\text{-}2)$$

where:

$p(t) =$ controller output

$\bar{p} =$ bias (steady-state) value

$K_c =$ controller gain (usually dimensionless)



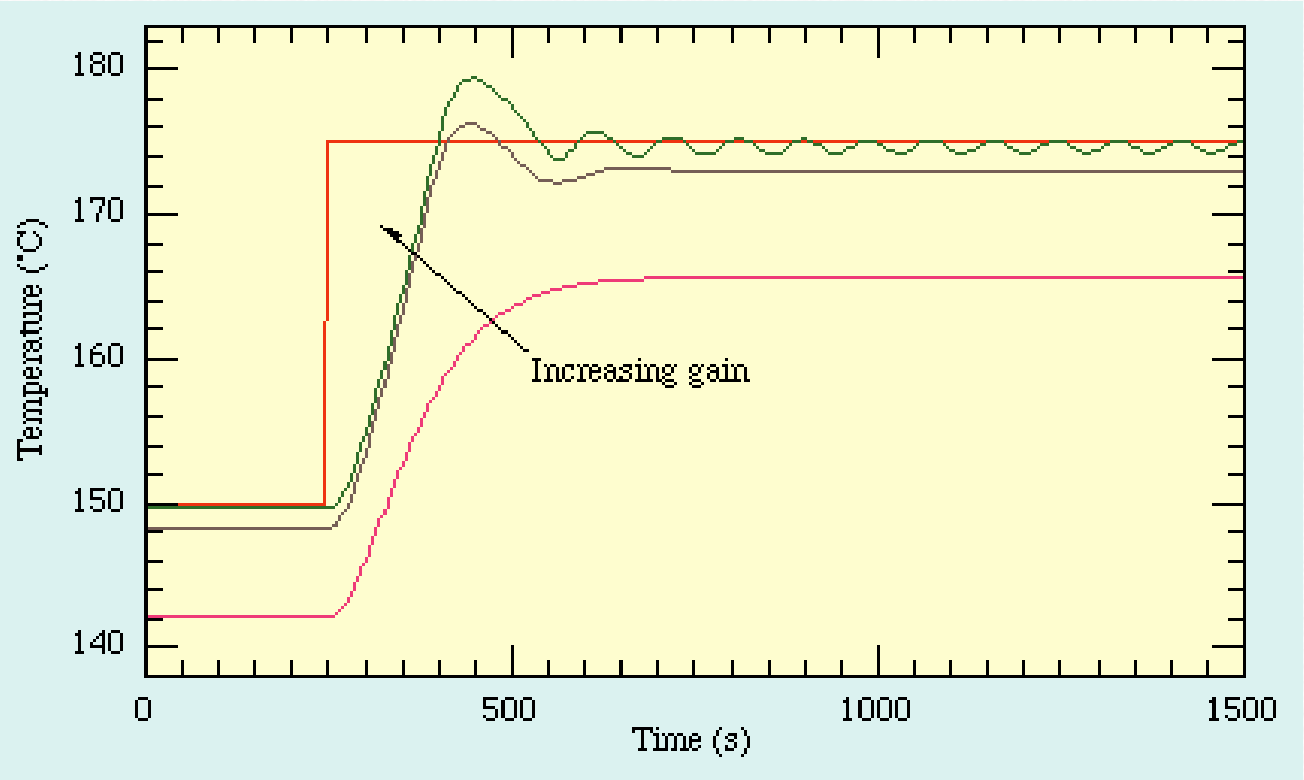
$$G(s) = \frac{E_o(s)}{E_i(s)} \quad \frac{R_4}{R_3}\,\frac{R_2}{R_1}$$

Basic Control Modes

Proportional Control

- Increasing gain approaches setpoint faster
- Can leads to overshoot, and even instability
- Steady-state offset

An inherent disadvantage of proportional-only control is that a steady-state error occurs after a set-point change or a sustained disturbance.



Basic Control Modes

Integral Control

For integral control action, the controller output depends on the integral of the error signal over time,

where τ_I , an adjustable parameter referred to as the integral time or reset time, has units of time.

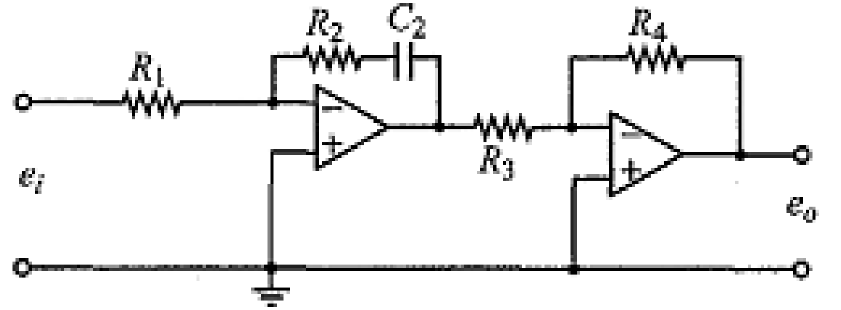
$$p(t) = \bar{p} + \frac{1}{\tau_I}\int_0^t e(t^*)dt^* \qquad (8\text{-}7)$$

Integral control action is widely used because it provides an important practical advantage, the elimination of offset. Consequently, integral control action is normally used in conjunction with proportional control as the *proportional-integral (PI)* controller:

$$p(t) = \bar{p} + K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)\,dt^*\right) \qquad (8\text{-}8)$$

Basic Control Modes

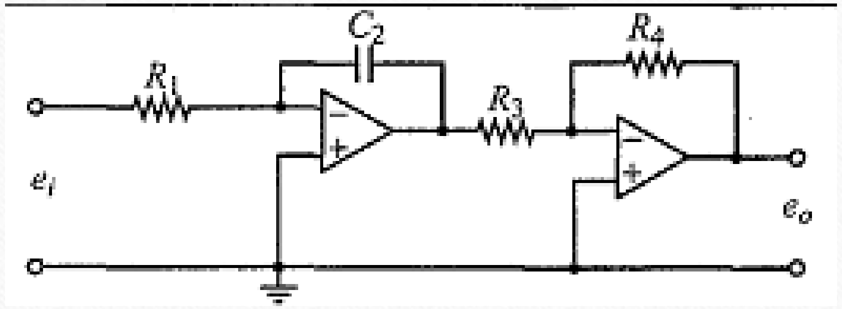
PI controller



$$G(s) = \frac{E_o(s)}{E_i(s)} \; \frac{R_4}{R_3} \, \frac{R_2}{R_1} \, \frac{R_2 C_2 s + 1}{R_2 C_2 s}$$

$$p(t) = \bar{p} + K_c \left(e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) \, dt^* \right) \qquad (8\text{-}8)$$

I controller



$$G(s) = \frac{E_o(s)}{E_i(s)} \; \frac{R_4}{R_3} \, \frac{1}{R_1 C_2 s}$$

$$p(t) = \bar{p} + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* \qquad (8\text{-}7)$$

Basic Control Modes

Integral Control

Reset Windup

- An inherent disadvantage of integral control action is a phenomenon known as *reset windup* or *integral windup*.
- Recall that the integral mode causes the controller output to change as long as $e(t^*) \neq 0$ in Eq. 8-8.
- When a sustained error occurs, the integral term becomes quite large and the controller output eventually saturates.
- Further buildup of the integral term while the controller is saturated is referred to as reset windup or *integral windup.*

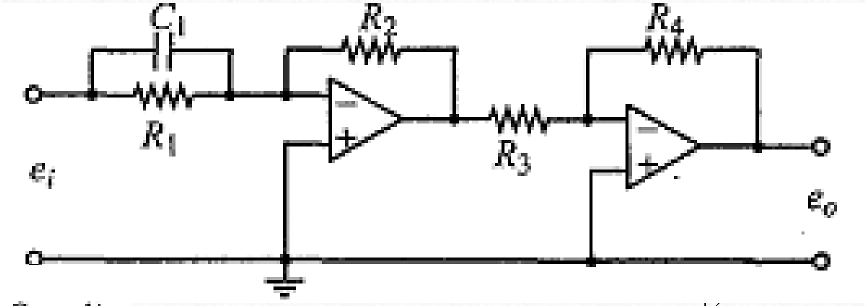
Basic Control Modes

Derivative Control

The function of derivative control action is to anticipate the future behavior of the error signal by considering its rate of change.

- The anticipatory strategy used by the experienced operator can be incorporated in automatic controllers by making the controller output proportional to the rate of change of the error signal or the controlled variable.
- Thus, for *ideal* derivative action,

$$p(t) = \bar{p} + \tau_D \frac{de(t)}{dt} \qquad (8\text{-}10)$$

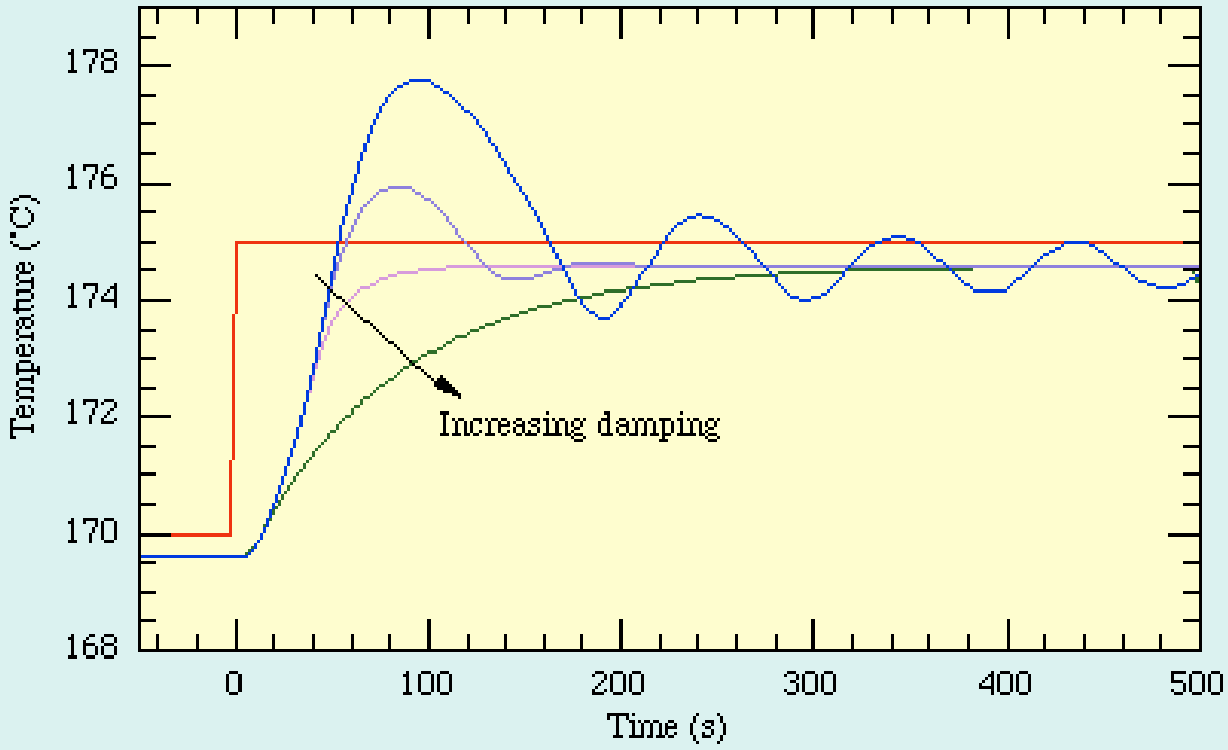


$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{R_4}{R_3}\frac{R_2}{R_1}(R_1 C_1 s + 1)$$

Basic Control Modes

Derivative Control

- Damping fights oscillation and overshoot
- But it’s vulnerable to noise



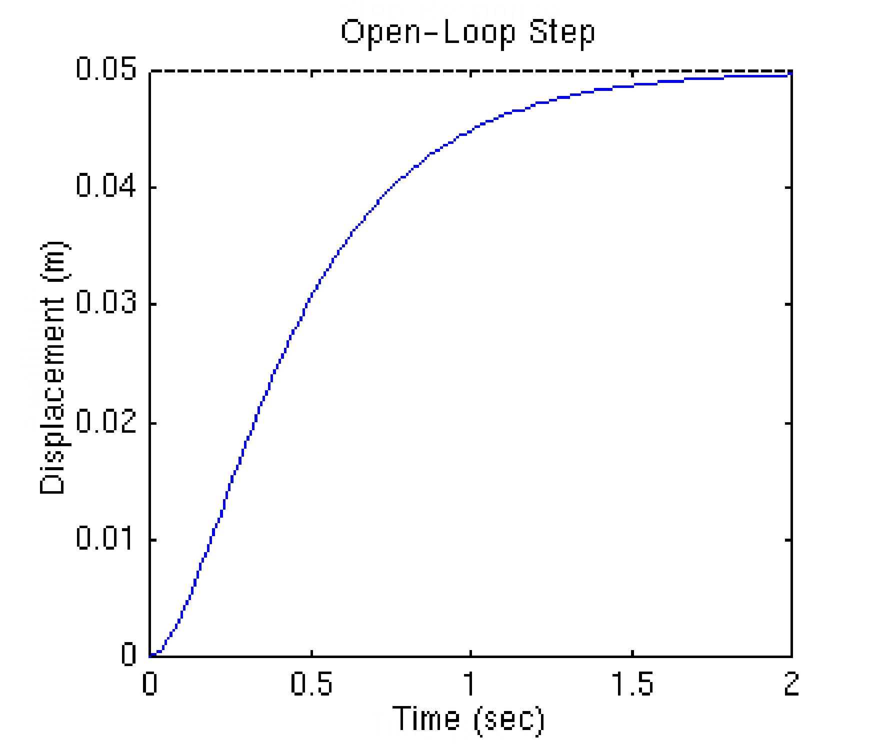
- Unfortunately, the ideal proportional-derivative control algorithm is *physically unrealizable* because it cannot be implemented exactly.

Basic Control Modes

Open-Loop Control - Example

$$G(s) = \frac{1}{s^2 + 10s + 20}$$

```
num=1;
den=[1 10 20];
step(num,den)
```



Basic Control Modes

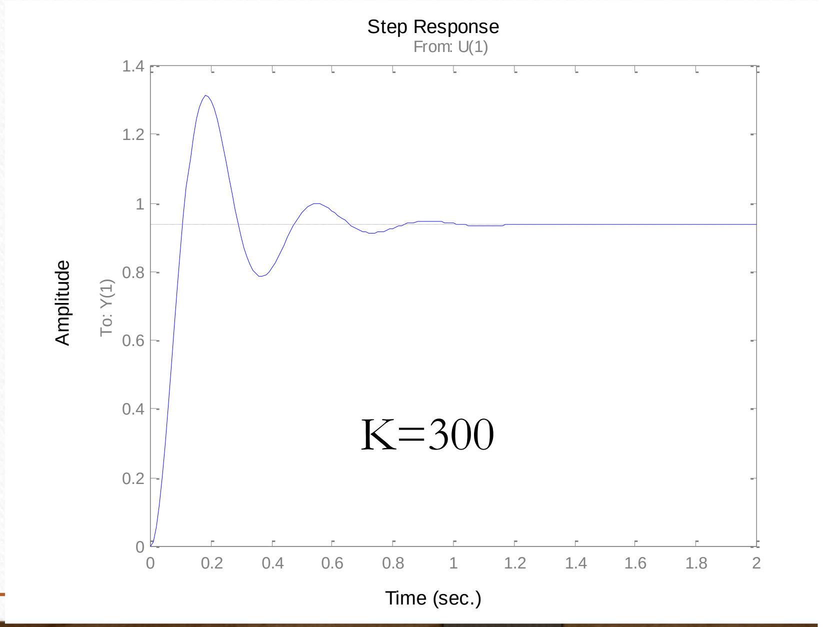
Proportional Control - Example

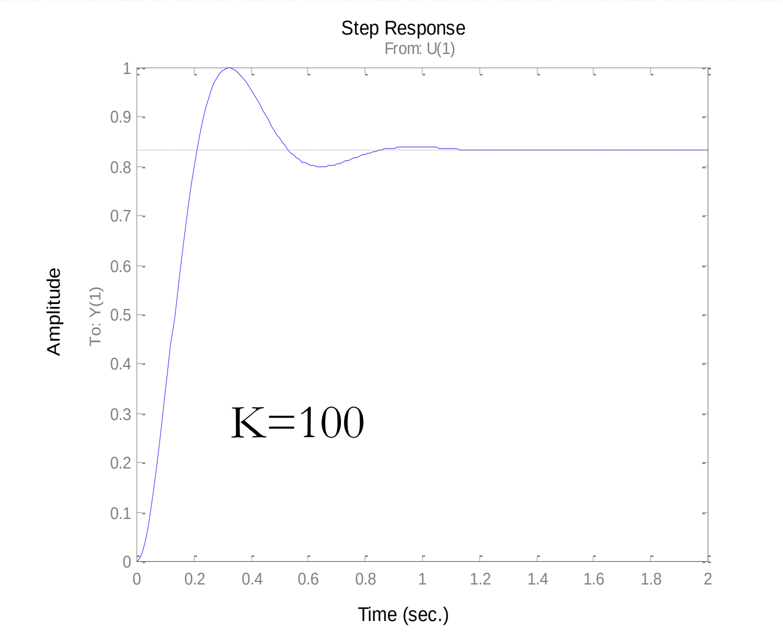
The proportional controller (Kp) reduces the rise time, increases the overshoot, and reduces the steady-state error.

$$T(s) = \frac{Kp}{s^2 + 10\cdot s + (20 + Kp)}$$

MATLAB Example

```
Kp=300;
num=[Kp];
den=[1 10 20+Kp];
t=0:0.01:2;
step(num,den,t)
```





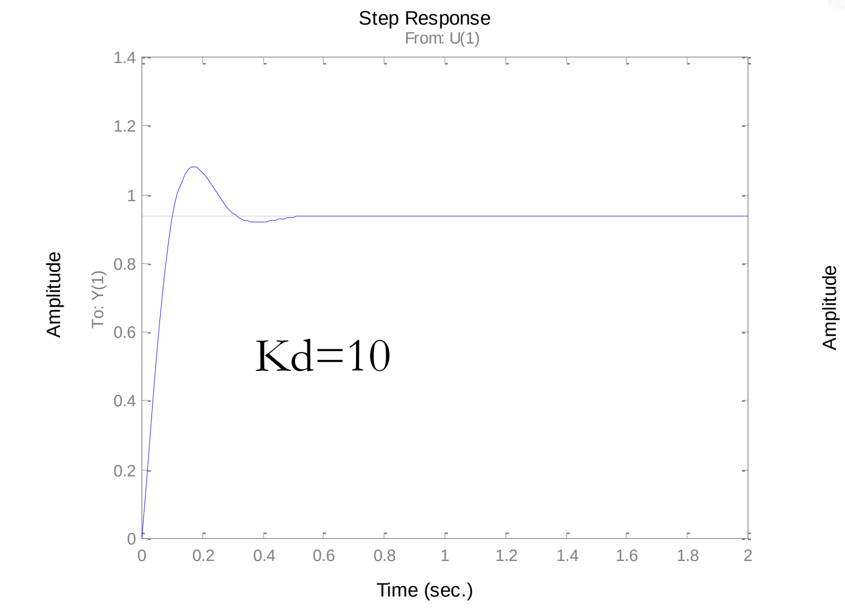
Basic Control Modes

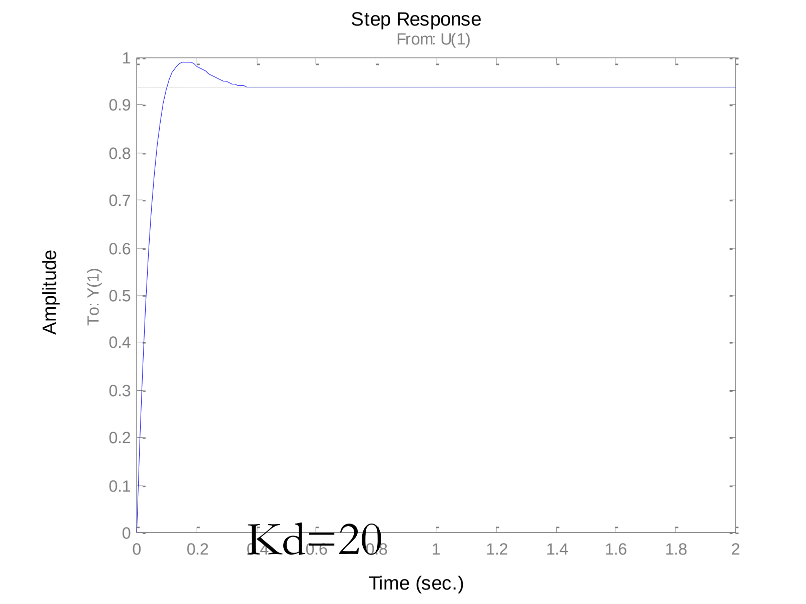
Proportional - Derivative - Example

The derivative controller (Kd) reduces both the overshoot and the settling time.
MATLAB Example

$$T(s) = \frac{Kd \cdot s + Kp}{s^2 + (10 + Kd) \cdot s + (20 + Kp)}$$

```
Kp=300;
Kd=10;
num=[Kd Kp];
den=[1 10+Kd 20+Kp];
t=0:0.01:2;
step(num,den,t)
```





Basic Control Modes

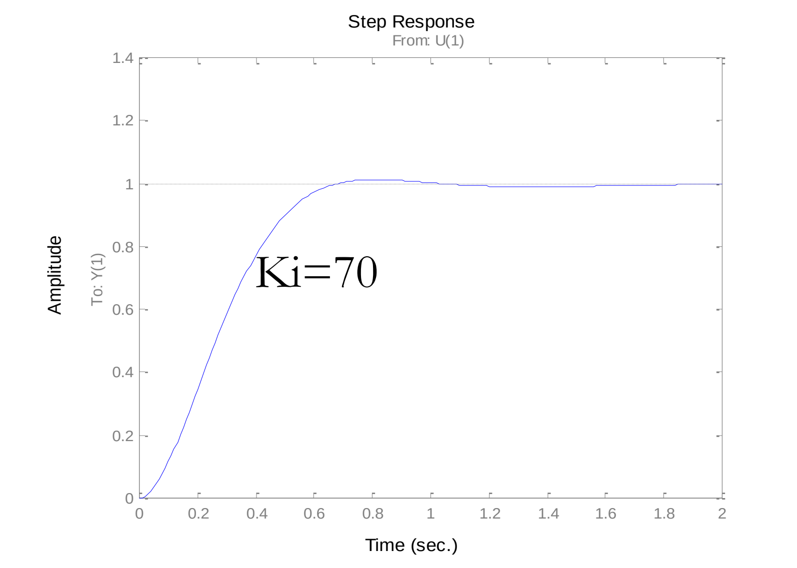
Proportional - Integral - Example

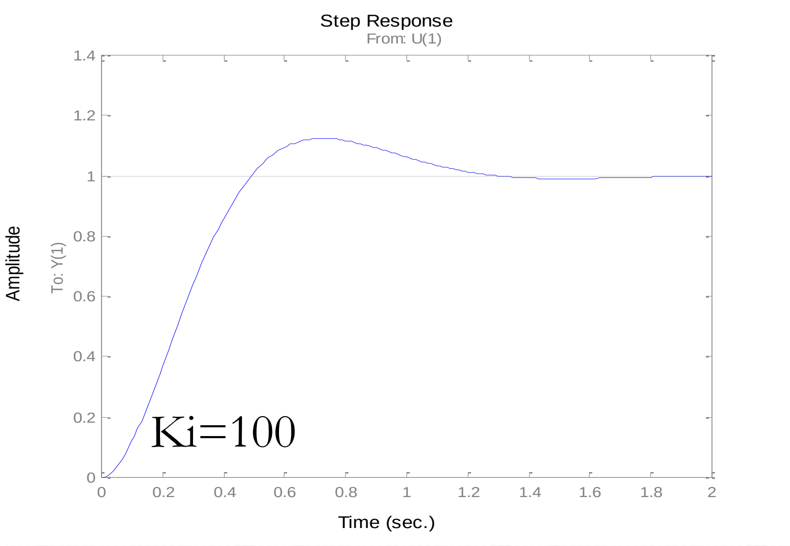
The integral controller (Ki) decreases the rise time, increases both the overshoot and the settling time, and eliminates the steady-state error

$$T(s) = \frac{Kp \cdot s + Ki}{s^3 + 10 \cdot s^2 + (20 + Kp) \cdot s + Ki}$$

MATLAB Example

```
Kp=30;

Ki=70;

num=[Kp Ki];

den=[1 10 20+Kp Ki];

t=0:0.01:2;

step(num,den,t)
```





Proportional-Integral-Derivative (PID) Control

The Characteristics of P, I, and D controllers

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

Proportional-Integral-Derivative (PID) Control

Tips for Designing a PID Controller

1. Obtain an open-loop response and determine what needs to be improved
2. Add a **proportional** control to improve the **rise time**
3. Add a **derivative** control to improve the **overshoot**
4. Add an **integral** control to eliminate the steady-state **error**
5. Adjust each of Kp, Ki, and Kd until you obtain a desired overall response.

PID Controllers

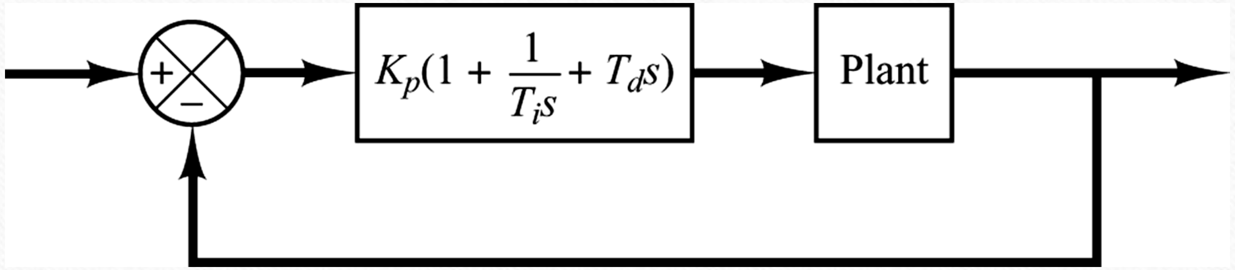
Tuning

- More than half of the industrial controllers in use today utilize PID or modified PID control schemes.
- Many different types of tuning :
  - ☞ Manual tuning on-site
  - ☞ On-line automatic tuning
  - ☞ Gain scheduling
- When the mathematical model of the plant is not known and therefore analytical design methods cannot be used, PID controls prove to be most useful.

PID Controllers

Design PID control

- Know mathematical model ☞ various design techniques
- Plant is complicated, can’t obtain mathematical model ☞ experimental approaches to the tuning of PID controllers



PID control of a plant.

Tuning of PID Controllers

Because of their widespread use in practice, we present below several methods for tuning PID controllers. Actually these methods are quite old and date back to the 1950's. Nonetheless, they remain in widespread use today.

In particular, we will study.

- *Ziegler-Nichols Reaction Curve Method*
- *Ziegler-Nichols Oscillation Method*

PID Controllers

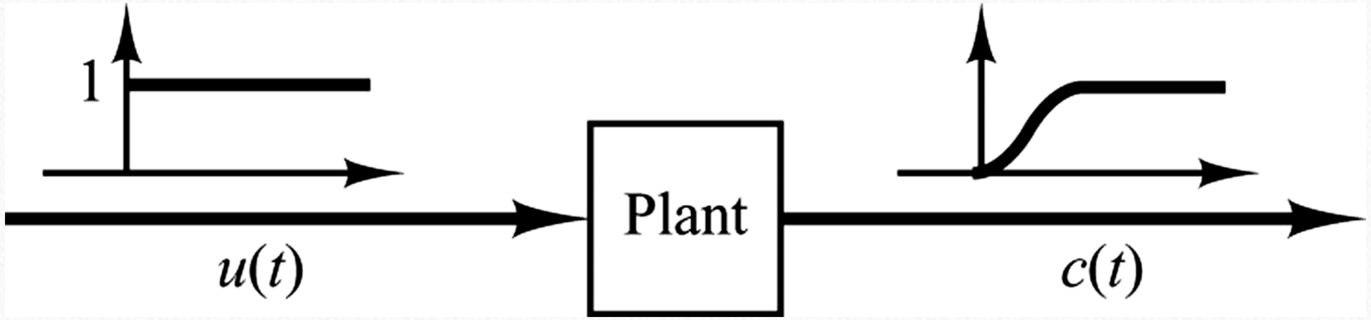
Ziegler-Nichols 1st Method of Tuning Rule

- We obtain experimentally the response of the plant to a unit-step input, as shown in Figure.

- The plant involves neither integrator(s) nor dominant complex-conjugate poles.

- This method applies if the response to a step input exhibits an S-shaped curve.

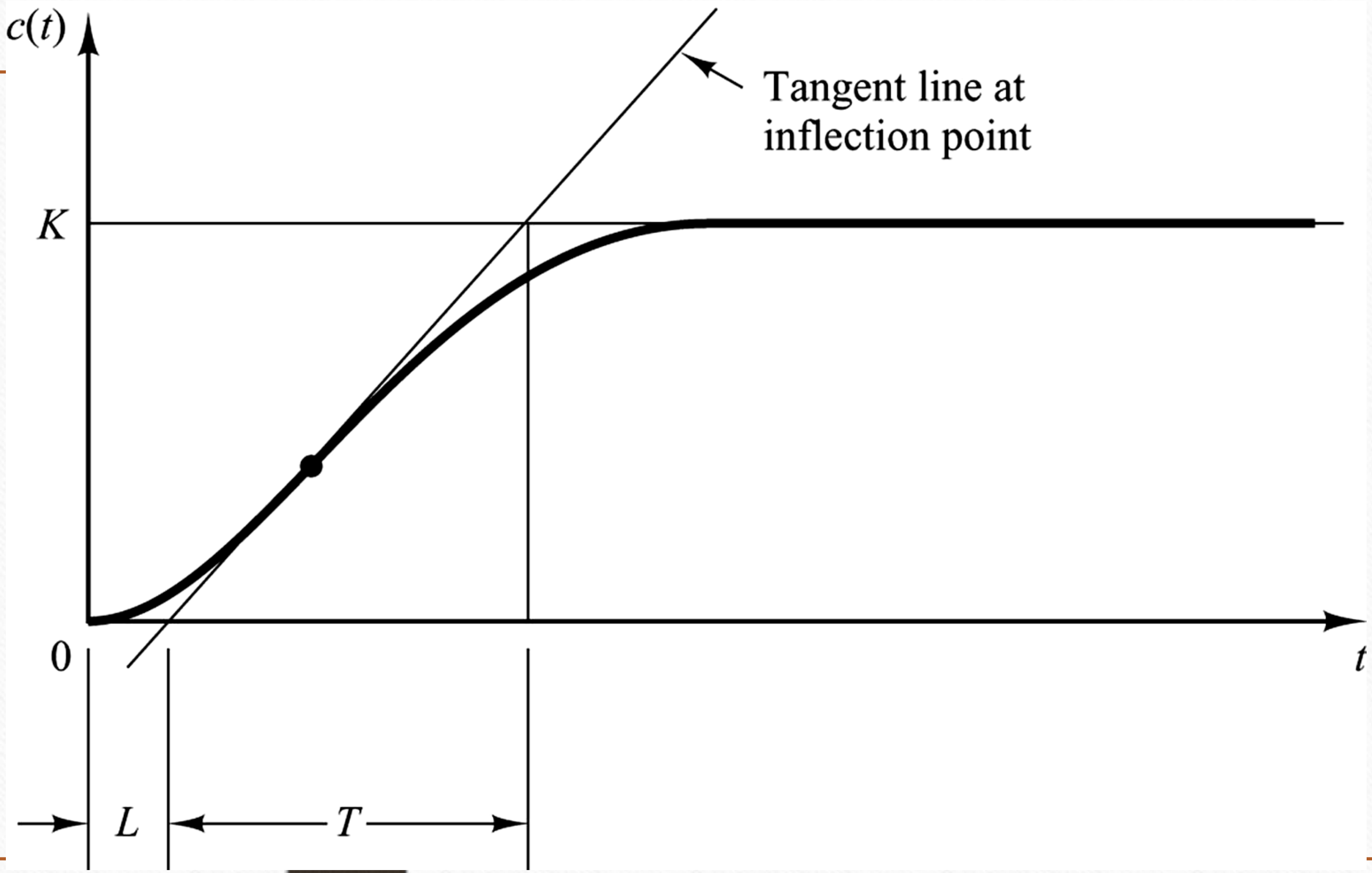
- Such step-response curves may be generated experimentally or from a dynamic simulation of the plant.



Unit-step response of a plant.

PID Controllers

S-shaped response curve.



L = delay time
T = time constant

PID Controllers

Transfer function:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Table 1 **Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)**

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

PID Controllers

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$
\begin{aligned}
G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\
&= 1.2\,\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\
&= 0.6T\,\frac{\left(s + \frac{1}{L}\right)^2}{s}
\end{aligned}
$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

PID Controllers

(٢)Ziegler-Nichols (Z-N) Oscillation Method

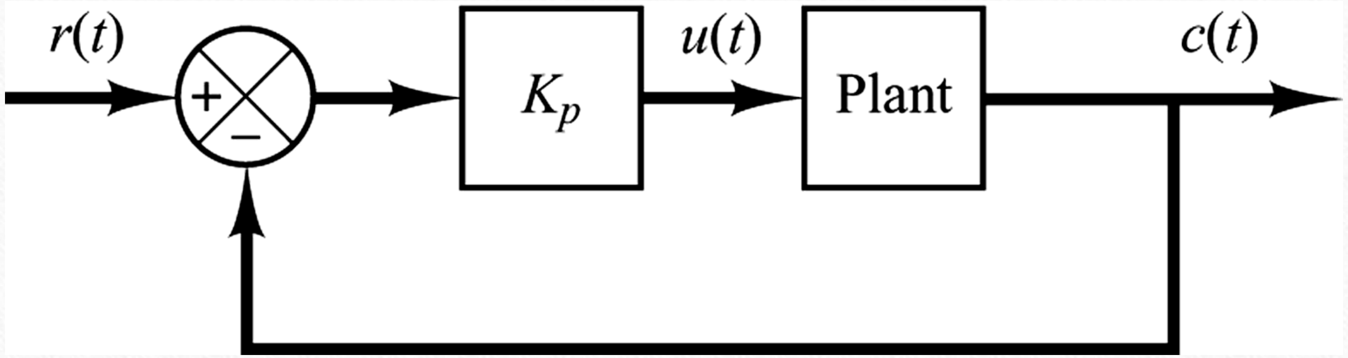
This procedure is only valid for open loop stable plants and it is carried out through the following steps

- Set the true plant under proportional control, with a very small gain.
- Increase the gain until the loop starts oscillating. Note that linear oscillation is required and that it should be detected at the controller output.

PID Controllers

Ziegler-Nichols 2nd Method of Tuning Rule

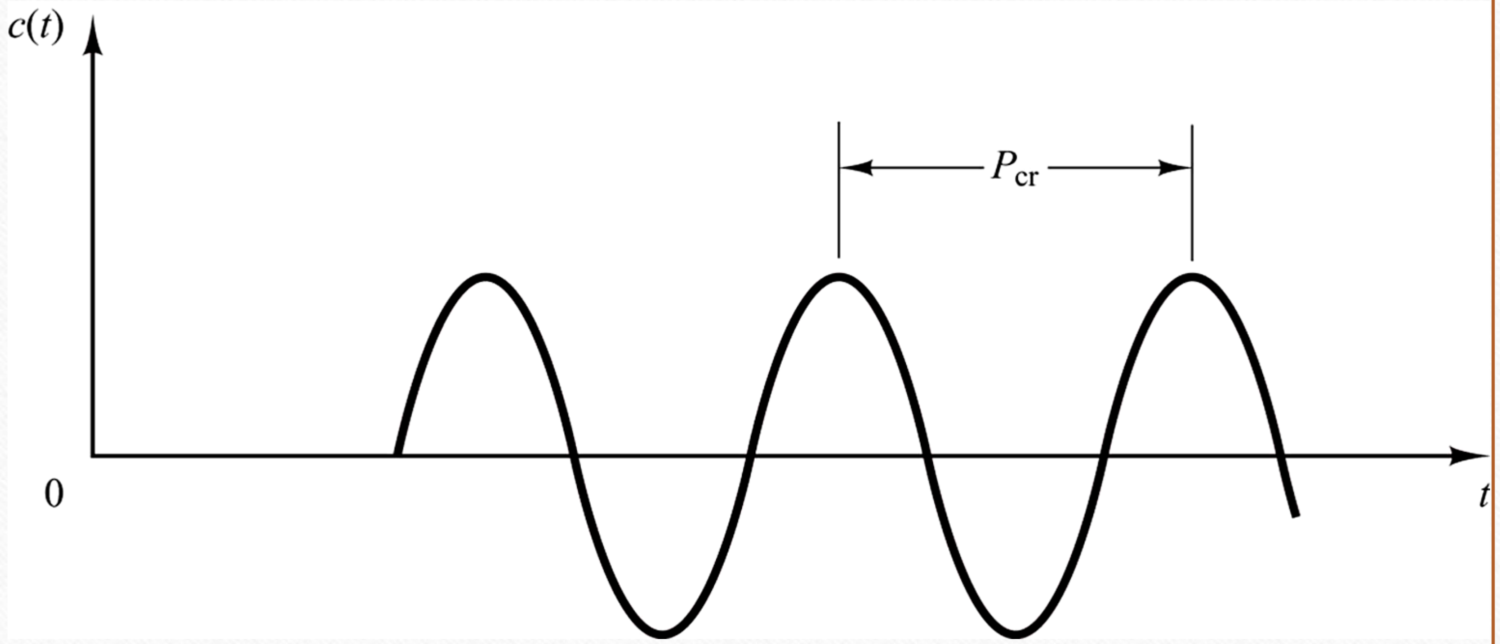
1. We first set $T_i = \infty$ and $T_d = 0$. Using the proportional control action only (see Figure).



Closed-loop system with a proportional controller.

PID Controllers

2. Increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations.



Sustained oscillation with period P_{cr}. (P_{cr} is measured in sec.)

PID Controllers

☞ Ziegler and Nichols suggested that we set the values of the parameters *K,, T,,* and *Td* according to the formula shown in Table 2.

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Table 2 **Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)**

PID Controllers

Note that if the system has a known mathematical model (such as the transfer function), then we can use the root-locus method to find the critical gain K_{cr} and the frequency of the sustained oscillations ω_{cr}, where $2\pi/\omega_{cr} = P_{cr}$. These values can be found from the crossing points of the root-locus branches with the $j\omega$ axis. (Obviously, if the root-locus branches do not cross the $j\omega$ axis, this method does not apply.)

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

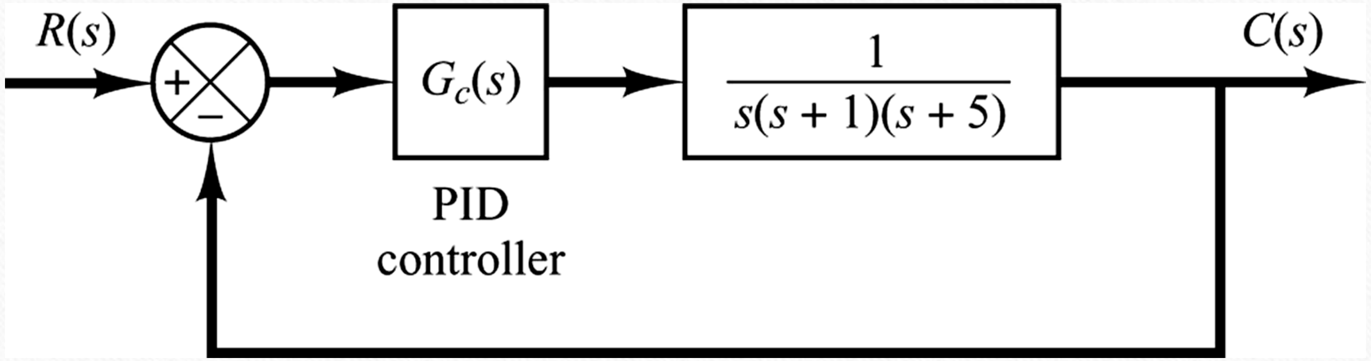
$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\ &= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

PID Controllers

Consider the control system shown in Figure 10–6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$



PID-controlled system.

PID Controllers

Since the plant has an integrator, we use the second method of Ziegler–Nichols tuning rules. By setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\boxed{\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5)+K_p}}$$

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$\boxed{s^3 + 6s^2 + 5s + K_p = 0}$$

PID Controllers

the Routh array becomes as follows:

$$\begin{array}{lcc} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30 - K_p}{6} & \\ s^0 & K_p & \end{array}$$

Examining the coefficients of the first column of the Routh table, we find that sustained oscillation will occur if $K_p = 30$. Thus, the critical gain K_{cr} is

$$K_{cr} = 30$$

With gain K_p set equal to K_{cr} (= 30), the characteristic equation becomes

$$\boxed{s^3 + 6s^2 + 5s + 30 = 0}$$

PID Controllers

To find the frequency of the sustained oscillation, we substitute $s = j\omega$ into this characteristic equation as follows:

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

or

$$6(5 - \omega^2) + j\omega(5 - \omega^2) = 0$$

from which we find the frequency of the sustained oscillation to be $\omega^2 = 5$ or $\boxed{\omega = \sqrt{5}}$. Hence, the period of sustained oscillation is

$$\boxed{P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} = 2.8099}$$

Referring to Table 10–2, we determine K_p, T_i, and T_d as follows:

$$\boxed{\begin{aligned} K_p &= 0.6K_{cr} = 18 \\ T_i &= 0.5P_{cr} = 1.405 \\ T_d &= 0.125P_{cr} = 0.35124 \end{aligned}}$$

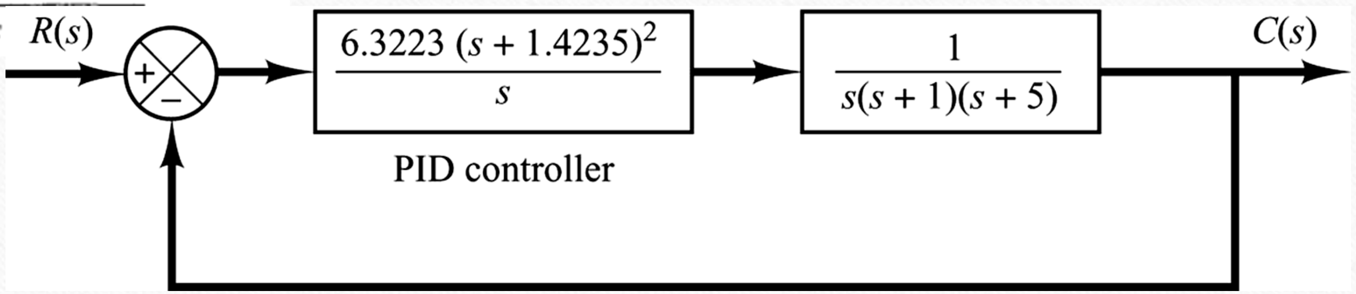
PID Controllers

The transfer function of the PID controller is thus

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right)$$

$$= \frac{6.3223(s + 1.4235)^2}{s}$$



Block diagram of the system with PID controller designed by use of the Ziegler–Nichols tuning rule (second method).

PID Controllers

Next, let us examine the unit-step response of the system. The closed-loop transfer function $C(s)/R(s)$ is given by

$$\frac{C(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$

MATLAB Program 10–1

```
% ---------- Unit-step response ----------
num = [0 0 6.3223 18 12.811];
den = [1 6 11.3223 18 12.811];
step(num,den)
grid
title('Unit-Step Response')
```

PID Controllers

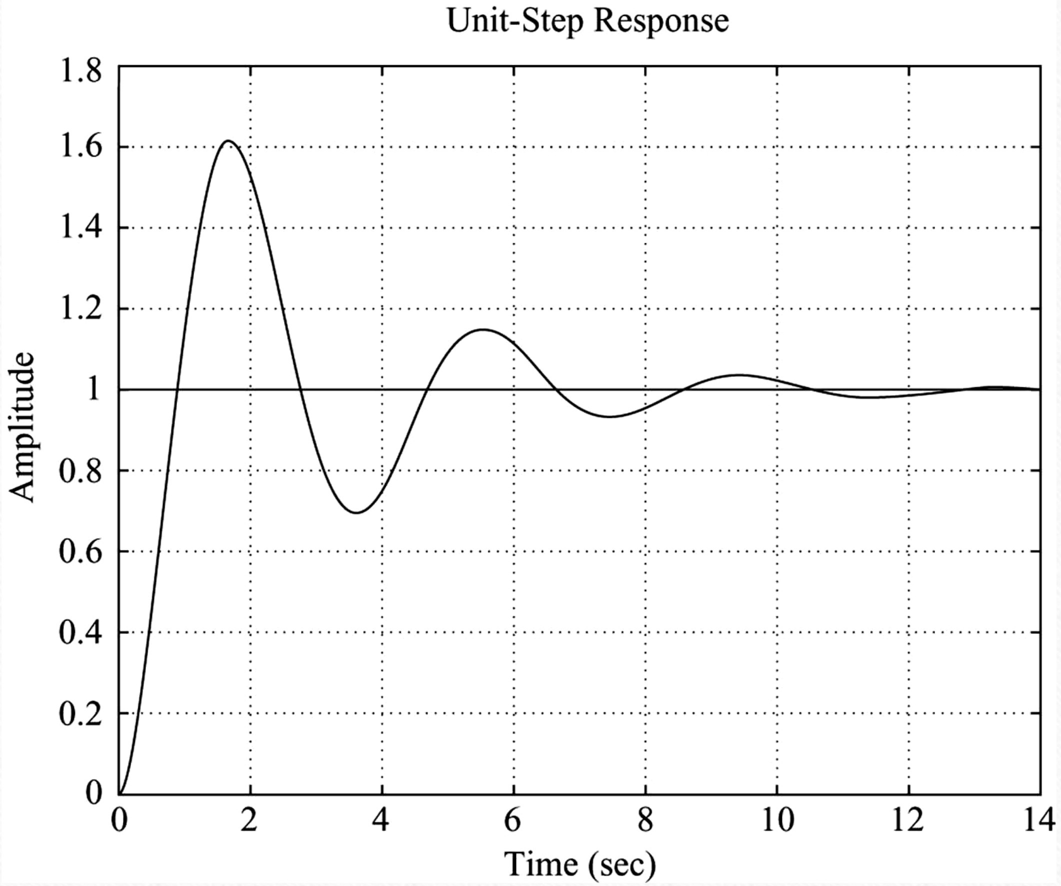


Figure 10-8 **Unit-step response curve of PID-controlled system designed by use of the Ziegler–Nichols tuning rule (second method).**

This is a physical demonstration of a PID controller controlling the angular position of the shaft of a DC motor. It was designed as a teaching tool to show the effects of proportional, integral, and derivative control schemes as well as the effect of saturation, anti-windup, and controller update rate on stability, overshoot, and steady state error.   Enjoy!

Gregory Holst
December 2015
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When PID Control is Used

- PID control works well on SISO systems of 2nd Order, where a desired Set Point can be supplied to the system control input
- PID control handles step changes to the Set Point especially well:
  - Fast Rise Times
  - Little or No Overshoot
  - Fast settling Times
  - Zero Steady State Error
- PID controllers are often fine tuned on-site, using established guidelines

PID Control

- A closed loop (feedback) control system, generally with Single Input-Single Output (SISO)
- A portion of the signal being fed back is:
  - Proportional to the signal (**P**)
  - Proportional to integral of the signal (**I**)
  - Proportional to the derivative of the signal (**D**)

Basic Control Modes

Proportional Control

The key concepts behind proportional control are the following:

1. The controller gain can be adjusted to make the controller output changes as sensitive as desired to deviations between set point and controlled variable;

2. the sign of K_c can be chose to make the controller output increase (or decrease) as the error signal increases.

For proportional controllers, bias $\bar{p}$ can be adjusted, a procedure referred to as *manual reset*.

Some controllers have a proportional band setting instead of a controller gain. The *proportional band PB* (in %) is defined as

$$PB \triangleq \frac{100\%}{K_c} \tag{8-3}$$

Basic Control Modes

Proportional Control

In order to derive the transfer function for an ideal proportional controller (without saturation limits), define a deviation variable $p'(t)$ as

$$p'(t) \triangleq p(t) - \bar{p} \qquad \text{(8-4)}$$

Then Eq. 8-2 can be written as

$$p'(t) = K_c e(t) \qquad \text{(8-5)}$$

The transfer function for proportional-only control:

$$\frac{P'(s)}{E(s)} = K_c \qquad \text{(8-6)}$$

An inherent disadvantage of proportional-only control is that a steady-state error occurs after a set-point change or a sustained disturbance.

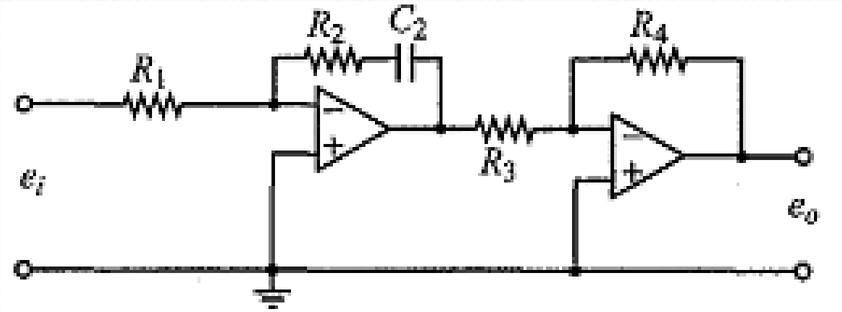
Basic Control Modes

Integral Control

The corresponding transfer function for the PI controller in Eq. 8-8 is given by

Some commercial controllers are calibrated in terms of $1/\tau_I$ (repeats per minute) rather than τ_I (minutes, or minutes per repeat).

$$\frac{P'(s)}{E(s)} = K_c\left(1+\frac{1}{\tau_I s}\right) = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right) \qquad (8\text{-}9)$$



$$G(s) = \frac{E_o(s)}{E_i(s)} \quad \frac{R_4}{R_3}\ \frac{R_2}{R_1}\ \frac{R_2C_2s+1}{R_2C_2s}$$

Basic Control Modes

Derivative Control

where τ_D, the derivative time, has units of time.

For example, an ideal PD controller has the transfer function:

$$\frac{P'(s)}{E(s)} = K_c\left(1+\tau_D s\right) \tag{8-11}$$

- By providing anticipatory control action, the derivative mode tends to stabilize the controlled process.
- Unfortunately, the ideal proportional-derivative control algorithm in Eq. 8-10 is *physically unrealizable* because it cannot be implemented exactly.

Basic Control Modes

Derivative Control

- For analog controllers, the transfer function in (8-11) can be approximated by

$$\frac{P'(s)}{E(s)} = K_c\left(1+\frac{\tau_D s}{\alpha\tau_D s+1}\right) \qquad (8\text{-}12)$$

where the constant α typically has a value between 0.05 and 0.2, with 0.1 being a common choice.

- In Eq. 8-12 the derivative term includes a *derivative mode filter* (also called a *derivative filter*) that reduces the sensitivity of the control calculations to high-frequency noise in the measurement.

PID Controllers

Ziegler-Nichols Rules for Tuning PID Controllers

-Ziegler and Nichols proposed rules for determining values of the proportional gain K_p, integral time T_i, and derivative time T_d based on the transient response characteristics of a given plant.

-Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experiments on the plant.

-Such rules suggest a set of values of K_p, T_i, and T_d that will give a stable operation of the system. However, the resulting system may exhibit a large maximum overshoot in the step response, which is unacceptable.

- We need series of fine tunings until an acceptable result is obtained.

PID Controllers

(١) Reaction Curve Based Methods

A linearized quantitative version of a simple plant can be obtained with an open loop experiment, using the following procedure:

1. With the plant in open loop, take the plant manually to a normal operating point. Say that the plant output settles at $y(t) = y_0$ for a constant plant input $u(t) = u_0$.
2. At an initial time, t_0, apply a step change to the plant input, from u_0 to u_∞ (*this should be in the range of 10 to 20% of full scale*).